$$f = f_0 (1 + g_1)$$

$$g_1 = -\frac{\tau_{MFP}}{k_B T} u_{\alpha\beta}\left(\delta v_\alpha \delta v_\beta - \frac{\delta v^2}{3}\delta_{\alpha\beta}\right) - \frac{\tau_{MFP}}{T}\delta v_\alpha (\partial_{q_\alpha} T)\left(\frac{m}{2k_B T}\delta \vec{u}^2 - \frac{5}{2}\right)$$

$$\langle O \rangle_f = \frac{1}{n}\int d^3\vec{p}\, f(\vec{q},\vec{p},t)\, O(\vec{q},\vec{p})$$
$$= \frac{1}{n}\int d^3\vec{p}\,(1+g_1)\, O(\vec{q},\vec{p})\, f_0(\vec{q},\vec{p},t)$$
$$= \langle (1+g_1)\, O\rangle_{f_0}$$ we can use this to compute $P^1_{\alpha\beta}$

- Pressure

$$P_{\alpha\beta} = \underbrace{m n \langle \delta v_\alpha \delta v_\beta \rangle_f = m n \langle \delta v_\alpha \delta v_\beta\rangle_0}_{\text{leading order}} + m n \langle \delta v_\alpha \delta v_\beta g_1\rangle_0$$

$$m \cancel{n}\left[\frac{1}{\cancel{n}}\int d^3\vec{p}\, f_0(\vec{q},\vec{p},t)\,\delta v_\alpha \delta v_\beta\right] =$$

$$= \frac{m n}{(2\pi m k_B T)^{3/2}} m^3 \int d^3\delta v\, e^{-\frac{m\delta\vec{u}^2}{2k_BT}}\,\delta v_\alpha^2\,\delta_{\alpha\beta}$$

$$= \frac{m n}{(2\pi m k_B T)^{3/2}} m^3 \delta_{\alpha\beta}\left(\int d\delta v_\alpha\, \delta v_\alpha^2\, e^{-\frac{m\delta v_\alpha^2}{2k_BT}} \underbrace{\int d\delta v_\beta\, e^{-\frac{m\delta v_\beta^2}{2k_BT}}\int d\delta v_\gamma\, e^{-\frac{m\delta v_\gamma^2}{2k_BT}}}_{\left(\frac{\pi\, 2k_BT}{m}\right)}\right)$$

$$= \frac{\cancel{m} n m^3}{(2\pi m k_B T)^{3/2}}\delta_{\alpha\beta}\left(\frac{\pi \cancel{2}k_BT}{\cancel{m}}\right)\frac{1}{\cancel{2}}\frac{\sqrt{\pi}}{m^{3/2}}(2k_BT)^{3/2}\cdot$$

$$= \frac{n \cancel{m^3}(2\pi)^{3/2}(k_BT)^{5/2}}{(2\cancel{\pi} k_BT)^{3/2}\cancel{\pi^3}} = \underline{n k_B T}\,\delta_{\alpha\beta}$$ ☺

$$P_{\alpha\beta}^{1} = n k_B T \delta_{\alpha\beta} + M n \langle \delta v_\alpha \delta v_\beta g_1 \rangle_0 =$$

$$= -\frac{M^2 n \tau_{MFP}}{k_B T} u_{\gamma\delta} \Big(\underbrace{\langle \delta v_\alpha \delta v_\beta \delta v_\gamma \delta v_\delta \rangle_0}_{A} - \frac{\delta_{\gamma\delta}}{3} \underbrace{\langle \delta v_\alpha \delta v_\beta \delta v_\varepsilon \delta v_\varepsilon \rangle_0}_{B} \Big)$$

(A) $$u_{\gamma\delta} \langle \delta v_\alpha \delta v_\beta \delta v_\gamma \delta v_\delta \rangle_0 = \sum_{\gamma,\delta} \Big[u_{\gamma\delta} \underbrace{\langle \delta v_\alpha \delta v_\beta \rangle_0}_{\delta_{\alpha\beta}} \underbrace{\langle \delta v_\gamma \delta v_\delta \rangle_0}_{\delta_{\gamma\delta}} + \overset{u_{\gamma\delta}}{\underbrace{\langle \delta v_\alpha \delta v_\gamma \rangle}_{\delta_{\alpha\gamma}}} \underbrace{\langle \delta v_\beta \delta v_\delta \rangle}_{\delta_{\beta\delta}} + \overset{u_{\gamma\delta}}{\underbrace{\langle \delta v_\alpha \delta v_\delta \rangle}_{\delta_{\alpha\delta}}} \underbrace{\langle \delta v_\beta \delta v_\gamma \rangle}_{\delta_{\beta\gamma}} \Big] = 3 u_{\gamma\gamma} \delta_{\alpha\beta} \left(\frac{k_B T}{M}\right)^2 + u_{\alpha\beta} \left(\frac{k_B T}{M}\right)^2 + u_{\alpha\beta} \left(\frac{k_B T}{M}\right)^2$$

divergence $= \partial_\gamma u_\gamma$

$$= \left(3 u_{\gamma\gamma} \delta_{\alpha\beta} + 2 u_{\alpha\beta} \right) \left(\frac{k_B T}{M}\right)^2$$

(B) $$-u_{\gamma\delta} \frac{\delta_{\gamma\delta}}{3} \Big(\langle \delta v_\alpha \delta v_\beta \delta v_\varepsilon \delta v_\varepsilon \rangle_0 \Big)$$

$$= \left(\frac{1}{3} \sum_{\gamma,\delta} u_{\gamma\delta} \delta_{\gamma\delta} \right) \left(\sum_\varepsilon \langle \delta v_\alpha \delta v_\beta \delta v_\varepsilon \delta v_\varepsilon \rangle_0 \right) =$$

$$= 3 u_{\gamma\gamma} \Big[\sum_\varepsilon \Big(\overbrace{\langle \delta v_\alpha \delta v_\beta \rangle_0}^{\delta_{\alpha\beta}} \overbrace{\langle \delta v_\varepsilon \delta v_\varepsilon \rangle_0}^{\delta_{\varepsilon\varepsilon}} + \underbrace{\langle \delta v_\alpha \delta v_\varepsilon \rangle_0}_{\delta_{\alpha\varepsilon}} \underbrace{\langle \delta v_\beta \delta v_\varepsilon \rangle_0}_{\delta_{\beta\varepsilon}} + \underbrace{\langle \delta v_\alpha \delta v_\varepsilon \rangle_0}_{\delta_{\alpha\varepsilon}} \underbrace{\langle \delta v_\beta \delta v_\varepsilon \rangle_0}_{\delta_{\beta\varepsilon}} \Big) \Big]$$

$$= \underbrace{3 u_{\gamma\gamma}}_{\partial_\gamma u_\gamma} \Big[3 \delta_{\alpha\beta} + 2 \delta_{\alpha\beta} \Big]$$

$$= \frac{5}{3} \partial_\gamma u_\gamma \delta_{\alpha\beta}$$

ok

a check

$$\frac{M^3 \cancel{n}}{(2\pi M k_B T)^{3/2}} \frac{1}{\cancel{n}} \int d^3 \delta v \, e^{-\frac{M \delta v^2}{2 k_B T}} \delta v_\alpha^2$$

$$= \frac{M^3 n}{(2\pi M k_B T)^{3/2}} \left(\frac{\pi \cancel{2} k_B T}{M} \right) \frac{1}{\cancel{2}} \frac{\sqrt{\pi}}{M^{3/2}} (2 k_B T)^{3/2}$$

$$= \left(\frac{k_B T}{M} \right) \quad \text{ok}$$

to remember:

$$\int_{-\infty}^{+\infty} dx \, e^{-a x^2} = \sqrt{\frac{\pi}{a}}$$

$$\frac{d}{da} \int_{-\infty}^{+\infty} dx \, e^{-a x^2} = \int_{-\infty}^{+\infty} dx \, (-x)^2 e^{-a x^2}$$

$$= -\frac{1}{2} \frac{\sqrt{\pi}}{a^{3/2}}$$

All in all:

$$P^1_{\alpha\beta} = n k_B T \delta_{\alpha\beta} - \frac{\cancel{m^2} n \tau_{mFP}}{\cancel{k_B T}} \left(\frac{k_B T}{m}\right)^{\cancel{2}} \left[\partial_\gamma u_\gamma \delta_{\alpha\beta} + 2u_{\alpha\beta} - \frac{5}{3}\partial_\gamma u_\gamma \delta_{\alpha\beta}\right]$$

$$= n k_B T \delta_{\alpha\beta} - \underbrace{n k_B T \tau_{mFP}}_{\text{viscosity coefficient}} \left(\partial_\gamma u_\gamma \delta_{\alpha\beta} + 2u_{\alpha\beta} - \frac{5}{3}\partial_\gamma u_\gamma \delta_{\alpha\beta}\right)$$

Heath flow:

$$h_\alpha = \frac{nm}{2}\langle \delta v_\alpha \delta\vec{v}^2 (1+g_1)\rangle_0 = \cancel{\frac{nm}{2}\langle \delta v_\alpha \delta\vec{v}^2\rangle_0}^{\,0} + \frac{nm}{2}\langle \delta v_\alpha \delta\vec{v}^2 g_1\rangle_0$$

$$= \frac{nm^2}{4k_BT}\left(-\frac{\tau_{mFP}}{T}\right)(\partial_{q_\beta} T)\underbrace{\langle \delta v_\alpha \delta\vec{v}^2 \delta v_\beta \delta\vec{v}^2\rangle}_{A} + \frac{nm}{4T} 5\,\tau_{mFP}(\partial_{q_\beta} T)\underbrace{\langle \delta v_\alpha \delta v_\beta \delta\vec{v}\rangle}_{B}$$

(A) $\langle \delta v_\alpha \delta v_\varepsilon \delta v_\varepsilon \delta v_\beta \delta v_\gamma \delta v_\gamma\rangle = \sum_{\varepsilon,\gamma}\langle \delta v_\alpha \delta v_\varepsilon^2 \delta v_\gamma^2 \delta v_\beta\rangle =$

first we take one of the two sums

$$\frac{2m!}{2^m m!} \overset{m=3}{=} \frac{6!}{2^3 3!} = \frac{720}{8\cdot 6} = 15$$

δ and ε are dummy indices

$$= \sum_{\varepsilon,\gamma}\delta_{\alpha\beta}\langle \delta v_\alpha^2 \delta v_\varepsilon^2 \delta v_\gamma^2\rangle = \delta_{\alpha\beta}\langle \delta v_\alpha^6\rangle_{\varepsilon=\gamma=\alpha} + 2\sum_{\varepsilon\neq\alpha}\delta_{\alpha\beta}\underbrace{\langle \delta v_\alpha^4 \delta v_\varepsilon^2\rangle_{\delta=\alpha}}$$

$$+ \sum_{\varepsilon\neq\alpha}\delta_{\alpha\beta}\underbrace{\langle \delta v_\alpha^2 \delta v_\varepsilon^4\rangle_{\varepsilon=\gamma}} + \sum_{\varepsilon\neq\alpha\neq\delta}\delta_{\alpha\beta}\underbrace{\langle \delta v_\alpha^2 \delta v_\varepsilon^2 \delta v_\gamma^2\rangle} =$$

2×2 possible coordinates $\varepsilon\neq\alpha$

$4\,\delta_{\alpha\beta}\langle \delta v_\alpha^4\rangle\langle \delta v_\varepsilon^2\rangle$

$= 12\,\delta_{\alpha\beta}\langle \delta v_\alpha^2\rangle\langle \delta v_\alpha^2\rangle\langle \delta v_\varepsilon^2\rangle$

2×2×3 combinatorics $(m=2)$

two coord. $2\langle \delta v_\alpha^2\rangle\langle \delta v_\varepsilon^4\rangle$

$$= \delta_{\alpha\beta}\left[15 + \right.$$

$6\langle \delta v_\alpha^2\rangle\langle \delta v_\varepsilon^2\rangle\langle \delta v_\varepsilon^2\rangle$

combinatorics

$2\,\delta_{\alpha\beta}\langle \delta v_\alpha^2\rangle\langle \delta v_\gamma^2\rangle\langle \delta v_\varepsilon^2\rangle$

$$= \delta_{\alpha\beta}\left[15 + 12 + 6 + 2\right]\left(\frac{k_BT}{m}\right)^3$$

$$- \delta_{\alpha\beta}\, 3\, 5 \left(\frac{k_B T}{m}\right)^3$$

(B) $\frac{n m^5}{4} \left(\frac{\tau_{MFP}}{T}\right) \left(\partial_{q_\beta} T(\vec{q}, t)\right) \langle \delta v_\alpha \, \delta v_\beta \, \delta \vec{v}^{\,2} \rangle$.

$$\sum_\gamma \langle \delta v_\alpha \delta v_\beta \delta v_\gamma^2 \rangle = \sum_\gamma \delta_{\alpha\beta} \langle \delta v_\alpha^2 \, \delta v_\gamma^2 \rangle = \delta_{\alpha\beta} \langle \delta v_\alpha^4 \rangle +$$

$$2\, \delta_{\alpha\beta} \langle \delta v_\alpha^2 \, \delta v_\gamma^2 \rangle_{\gamma \neq \alpha} = \left(3 \delta_{\alpha\beta} + 2 \delta_{\alpha\beta}\right) \left(\frac{k_B T}{m}\right)^2$$

All in all:

$$h_\alpha = \frac{n m^2}{k_B T\, 4} \left(-\frac{\tau_{MFP}}{T}\right) \left(\partial_{q_\beta} T\right) 3 5\, \delta_{\alpha\beta} \left(\frac{k_B T}{m}\right)^3 + \frac{n m^5}{4} \left(\frac{\tau_{MFP}}{T}\right) \left(\partial_{q_\beta} T\right) 5 \delta_{\alpha\beta} \left(\frac{k_B T}{m}\right)^2$$

$$= -\frac{n m}{4 T} \tau_{MFP} \left(\partial_{q_\beta} T\right) \delta_{\alpha\beta} \left[35 \left(\frac{k_B T}{m}\right)^2 - 25 \left(\frac{k_B T}{m}\right)^2 \right]$$

$$= -\frac{n m}{4 T} \tau_{MFP} \left(\partial_{q_\beta} T\right) \delta_{\alpha\beta}\, 10 \left(\frac{k_B T}{m}\right)^2 = \underbrace{-\frac{5}{2} \frac{n}{m} \tau_{MFP} k_B^2 T}_{\text{thermal conductivity}} \left(\partial_{q_\alpha} T\right)$$

$\square$

Hydrodynamic equations:

$$\begin{cases} D_t n = -n\,\partial_\alpha u_\alpha \\ M D_t u_\alpha = -\frac{1}{n}\partial_\beta P_{\alpha\beta} \\ D_t T = -\frac{2}{3 n k_B}\partial_\alpha h_\alpha - \frac{2}{3 n k_B} P_{\alpha\beta} u_{\alpha\beta} \end{cases}$$

First-order expressions:

$$P_{\alpha\beta} = \underbrace{n k_B T \delta_{\alpha\beta} + \frac{2}{3} n k_B T \delta_{\alpha\beta} \tau_{nFP} \partial_\gamma u_\gamma}_{\text{diagonal}} \underbrace{- 2 n k_B T \tau_{nFP} u_{\alpha\beta}}_{\text{off-diagonal}}$$

$$h_\alpha = -\frac{5}{2}\frac{n}{M}\tau_{nFP} k_B^2 T (\partial_{q_\alpha} T)$$

Let consider the linearised dynamics around

$n = n_0 + \delta n, \quad u = 0 + \delta u, \quad T = T_0 + \delta T$

$$D_t \delta n = -n_0 \partial_\alpha \delta u_\alpha$$

$$M D_t \delta u_\alpha = -\frac{1}{n_0}\partial_\beta \left[P^0_{\alpha\beta} + P^1_{\alpha\beta} \right]$$

$$= -\frac{1}{n_0}\partial_\beta \left[n_0 k_B \,\delta T \,\delta_{\alpha\beta} + k_B T_0 \delta_{\alpha\beta} \delta n \right] - \frac{1}{n_0}\partial_\beta \Big[\frac{2}{3}\mu_0 \delta_{\alpha\beta} \partial_\gamma u_\gamma$$

$$- \cancel{2}\mu_0 \frac{1}{\cancel{2}}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) \Big] = -\delta_{\alpha\beta} k_B \partial_\beta \delta T - \frac{k_B T_0}{n_0}\delta_{\alpha\beta}\partial_\beta \delta n +$$

$$-\frac{\mu_0}{n_0}\left[\frac{2}{3}\delta_{\alpha\beta}\partial_\beta\partial_\gamma u_\gamma - \partial_\beta\partial_\alpha u_\beta - \partial_\beta\partial_\beta u_\alpha \right] = -\delta_{\alpha\beta} k_B \partial_\beta \delta T$$

$$- k_B \frac{T_0}{n_0}\delta_{\alpha\beta}\partial_\beta \delta n - \frac{\mu_0}{n_0}\Big[\underbrace{\frac{2}{3}\partial_\alpha\partial_\beta u_\beta - \partial_\beta\partial_\alpha u_\beta}_{-\frac{1}{3}} - \delta_{\alpha\beta}\partial_\gamma\partial_\gamma u_\beta \Big]$$

$$\Rightarrow M D_t \delta u_\alpha = - k_B \partial_\alpha \delta T - \frac{k_B T_0}{n_0} \partial_\alpha \delta n + \frac{\mu_0}{n_0} \frac{1}{3} \partial_\beta \partial_\alpha u_\beta$$

$$+ \frac{\mu_0}{n_0} \partial_\gamma \partial_\gamma u_\beta \delta_{\alpha\beta}$$

$$D_t \delta T = + \frac{2}{3 n_0 k_B} \partial_\alpha^2 (k_0 \delta T) - \frac{2}{3 n_0 k_B} \underbrace{P_{\alpha\beta}^{(0)} u_{\alpha\beta}}_{n_0 k_B T \partial_\alpha u_\alpha}$$

$$D_t \delta n = - n_0 \partial_\alpha \delta u_\alpha$$

$$F(t, \vec{k}) = \int d\vec{q}\, e^{-i k \cdot q} F(\vec{q}, t)$$

$$\begin{cases} D_t \delta n = - n_0 \partial_\beta \delta u_\beta \\ M D_t \delta u_\alpha = - \frac{k_B T_0}{n_0} \partial_\alpha \delta n - k_B \partial_\alpha \delta T + \frac{\mu_0}{n_0} \left(\frac{1}{3} \partial_\alpha \partial_\beta + \delta_{\alpha\beta} \partial_\gamma \partial_\gamma \right) u_\beta \\ D_t \delta T = \frac{2 k_0}{3 n_0 k_B} \partial_\alpha \partial_\alpha \delta T - \frac{2}{3 \cancel{n_0} \cancel{k_B}} \cancel{n_0} \cancel{k_B} T_0 \partial_\alpha u_\alpha \\ \quad = \frac{2 k_0}{3 n_0 k_B} \partial_\alpha \partial_\alpha \delta T - \frac{2}{3} T_0 \partial_\alpha u_\alpha \end{cases}$$

$$D_t \delta \hat{n} = + n_0 i k_\alpha \delta \hat{u}_\alpha$$

$$D_t \delta \hat{u}_\alpha = + \frac{k_B T_0}{M n_0} i k_\alpha \delta \hat{n} + \frac{k_B i k_\alpha}{M} \delta \hat{T} - \frac{\mu_0}{n_0 M} \frac{1}{3} k_\alpha k_\beta \hat{u}_\beta - \frac{\mu_0}{n_0 M} \delta_{\alpha\beta} k_\gamma^2 \hat{u}_\beta$$

$$D_t \delta \hat{T} = \frac{-2 k_0}{3 n_0 k_B} k_\alpha^2 \delta \hat{T} + \frac{2 i}{3} T_0 k_\alpha u_\alpha$$

About microcanonical ensemble :

IDEAL GAS

infinitesimal volume in phase space

$$\tilde{\Omega}(E) = \int_{E \leq H(\{\vec{q}_i, \vec{p}_i\}) \leq E+\delta E} \prod_{i=1}^{N} \left(\frac{d^3\vec{q}_i \, d^3\vec{p}_i}{h^3} \right) = \left(\frac{V}{h^3}\right)^N \int_{E \leq \sum_i \frac{p_i^2}{2m} \leq E+\delta E} \prod_{i=1}^{N} d^3\vec{p}_i = \left(\frac{V}{h^3}\right)^N \left(\mathcal{V}(E+\delta E) - \mathcal{V}(E)\right)$$

$$\simeq \left(\frac{V}{h^3}\right)^N \mathcal{V}'(E)\,\delta E \qquad \text{with } \mathcal{V}(E) = \frac{\Omega_{S_0}(3N)\,(2mE)^{3N/2}}{3N}$$

$$\text{and solid angle} \quad \Omega_{S_0}(d) = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

Then, we first properly take into account indistinguishability by dividing by $N!$ and we compute the entropy :

$$\Rightarrow \quad S_m = N k_B \ln\left[\frac{eV}{h^3 N} \left(\frac{4\pi e m E}{3N} \right)^{3/2} \right]$$

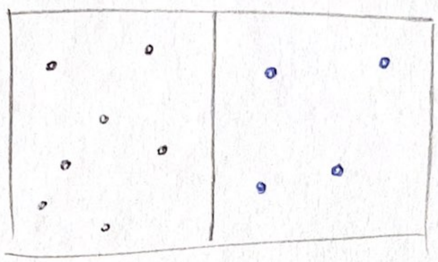
(in passing we have used Stirling approximation)

We also found : $E = \frac{3}{2} N k_B T$

$$\Rightarrow \quad \boxed{S_m = N k_B \ln\left[\frac{eV}{N} \left(\frac{2\pi e m k_B T}{h^2} \right)^{3/2} \right]}$$

Entropy of mixing

Initial state

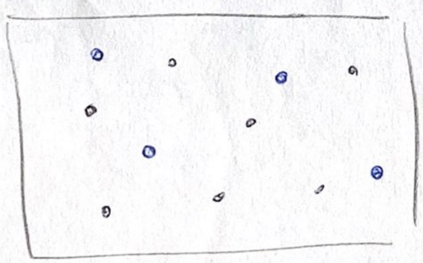


N_A	N_B
V	V
T	T
$m_A = m$	$m_B = m$

two distinguishable gases with same masses
in the same volume and
at the same temperature

What happens to the entropy
when I mix the two
gases?

Final state



$$\Delta S = S_A^{\text{final}} + S_B^{\text{final}} - S_A^{\text{initial}} - S_B^{\text{initial}}$$

I am setting $\lambda = \dfrac{h^2}{2\pi e\, m k_B T}$

$$= N_A k_B \ln\left[\frac{e2V}{N_A \lambda^3}\right] + N_B k_B \ln\left[\frac{e2V}{N_B \lambda^3}\right] - N_A k_B \ln\left[\frac{eV}{N_A \lambda^3}\right]$$

$$- N_B k_B \ln\left[\frac{eV}{N_B \lambda^3}\right] = \underline{(N_A + N_B)\, k_B \ln 2} = \Delta S_{mix}$$

entropy of mixing

if $N_A = N_B$

$\Delta S_{mix} = 2 N k_B \ln 2$

both gases here expanded!

Both individual entropies have increased because both gases have expanded.

Now, what happens if the two gases are indistinguishable? Since in this case the two gases are identical, the entropy of the final system can be written as:

$$S_{final} = (N_A + N_B) k_B \ln\left(\frac{e\,2V}{(N_A+N_B)\lambda^3}\right)$$

the initial state is the same!

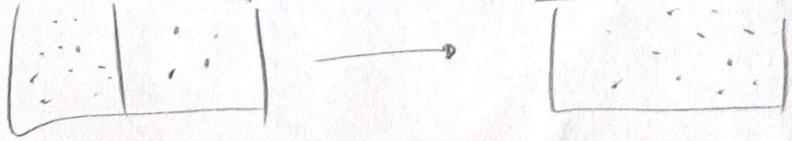
$$\Delta S = (N_A+N_B) k_B \ln\left(\frac{e\,2V}{(N_A+N_B)\lambda^3}\right) - N_A k_B \ln\left(\frac{e\,V}{N_A \lambda^3}\right) - N_B k_B \ln\left(\frac{e\,V}{N_B \lambda^3}\right)$$

$$= \boxed{N_A k_B \ln\left(\frac{2N_A}{N_A+N_B}\right) + N_B k_B \ln\left(\frac{2N_B}{N_A+N_B}\right)}$$

What if $N_A = N_B$

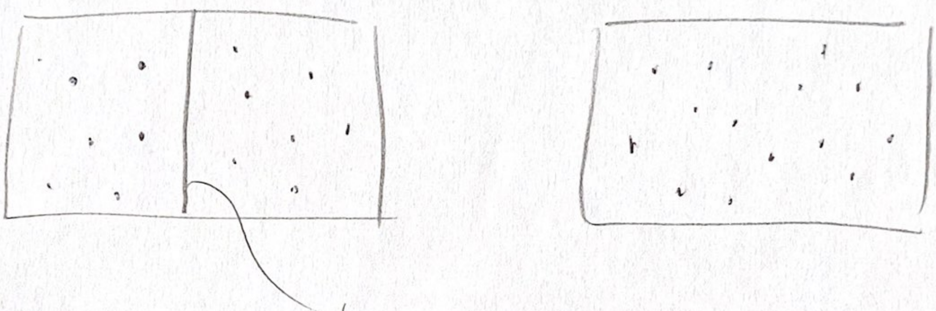
$\boxed{\Delta S = 0}$ No entropy increase!

So the entropy increases only if I go from a situation of two different densities to an homogeneous system (I have lost information on the system)



If $N_A = N_B$, despite the fact that the two gases expand, there is no gain in entropy!

As a matter of fact



having the barrier does not change anything...

NB in the previous case if $N_A = N_B$, still $\underline{\underline{\Delta S = 2N k_B \ln 2}}$

So the result depends on whether the particles are distinguishable or not!

Apparent paradox...

But it is not really a paradox, if you are able to distinguish them or not means that you have a different information on the system. In one case swapping two particles between containers does not make a difference, in the other case it does!!

TWO-LEVEL SYSTEM

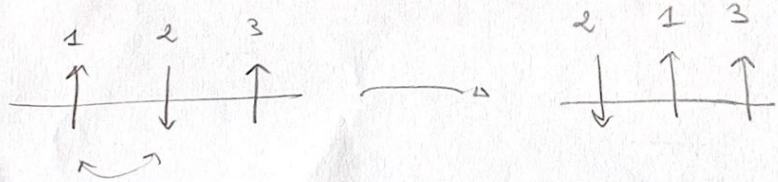
N atoms on a lattice which can only have two energy level $\begin{cases} \varepsilon \\ 0 \end{cases}$. Thus, $H = \sum_{i=1}^{N} \varepsilon_i = pN\varepsilon$

p — fraction of atoms with energy ε

$$\Omega(\underbrace{E = pN\varepsilon}_{\text{fixed energy}}) = \binom{N}{pN} = \underbrace{\frac{N!}{pN!\,(N-pN)!}}$$

this is counting in how many different ways I choose pN atoms among N of them.

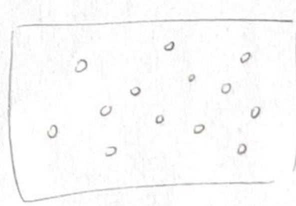
here we do not have to divide by $N!$ because we do distinguish atoms! — if this was an ideal gas $\rightarrow$ no difference!



these two configurations are clearly different!

So, what does the binomial coefficient actually count?

It has to do with distinguishability of configurations, not distinguishability of particles



instead in the ideal gas whatever two pair of particles I would switch $\rightarrow$ no difference!

ex $N = 3$, $p = 1$

$$\Omega(\varepsilon) = \binom{3}{1} = \frac{3!}{2!} = 3$$

			1 2 3
1	2	3	↑ ↓ ↓
1	3	2	↑ ↓ ↓
2	1	3	↓ ↑ ↓
2	3	1	↓ ↑ ↓
3	1	2	↓ ↓ ↑
3	2	1	↓ ↓ ↑

$\neq$ $\neq$

I take this as up

Deriving Stirling approximation:

$$N! = \Gamma(N+1) = \int_0^\infty dx\, x^N e^{-x} = \int_0^\infty dx\, e^{N\ln x - x}$$

change of variable: $y = x/N \quad dy = \frac{dx}{N}$

$$N! = N\int_0^\infty dy\, e^{N\ln Ny - Ny} = N\int_0^\infty dy\, e^{N[\ln y - y]}\, e^{N\ln N}$$

$$= N e^{N\ln N}\int_0^\infty dy\, e^{N(\ln y - y)}$$

Use saddle point method for $N \to \infty$

$$f(y) = \ln y - y$$

$$f'(y) = \frac{1}{y} - 1 = 0 \quad \Rightarrow \quad \text{maximize in} \quad y^* = 1$$

$$f''(y) = -\frac{1}{y^2} \quad \Rightarrow \quad f''(y^*) = -1$$

So:
$$N! \simeq N e^{N \ln N} \int_0^\infty dy \, e^{N\left[f(y^*) + \cancel{(y-y^*) f'(y^*)} + \frac{1}{2}(y-y^*)^2 \underbrace{f''(y^*)}_{=-1} + \dots\right.}$$

$$= N e^{N \ln N - N} \int_0^\infty dy \, e^{-\frac{N}{2}(y+1)^2} = N e^{N \ln N - N} \underbrace{\int_0^\infty d\bar{y} \, e^{-\frac{N}{2}\bar{y}^2}}_{\text{Gaussian integral!}}$$

$$= N e^{N \ln N - N} \sqrt{\frac{2\pi}{N}} = \sqrt{2\pi N} \, e^{N \ln N - N}$$